Letter

Magnetic dipole moment of the $\Delta(1232)$ in chiral perturbation theory

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Abstract. The magnetic dipole moment of the $\Delta(1232)$ is calculated in the framework of manifestly Lorentz-invariant baryon chiral perturbation theory in combination with the extended on-mass-shell renormalization scheme. As in the case of the nucleon, at leading order both isoscalar and isovector anomalous magnetic moments are given in terms of two low-energy constants. In contrast to the nucleon case, at next-to-leading order the isoscalar anomalous magnetic moment receives a (real) loop contribution. Moreover, due to the unstable nature of the $\Delta(1232)$, at next-to-leading order the isovector anomalous magnetic moment not only receives a real but also an imaginary loop contribution.

PACS. 12.39.Fe Chiral Lagrangians – 13.40.Em Electric and magnetic moments – 14.20.Gk Baryon resonances with $S = 0$

1 Introduction

The $\Delta(1232)$-resonance is the most prominent and best-studied nucleon resonance. It plays an important role in the phenomenological description of low- and medium-energy processes. This is due to the strong coupling of the $\Delta(1232)$ to the πN channel and the relatively small mass difference between the nucleon and the $\Delta(1232)$. The strong decay into a nucleon and a pion results in an extremely short lifetime and makes a precise determination of such a fundamental physical quantity as the magnetic dipole moment nontrivial. While the magnetic moments of (almost) stable particles may be determined by means of spin precession measurements, for unstable particles this is not possible. Here, one has to resort to indirect measurements making use of a superior physical reaction into which the electromagnetic interaction of the particle in question is embedded as a building block.

The magnetic moment of the $\Delta^{++}(1232)$ has been investigated experimentally by measuring the $\pi^+ p$ bremsstrahlung reaction [1,2] which has been analyzed within various theoretical frameworks [3–7]. The Particle Data Group only makes a rough estimate of the range the moment is expected to lie within, $\mu_{\Delta^{++}} =$ $(3.7$–$7.5)\,\mu_N$ [8][1], while $SU(6)$ symmetry predicts for a member of the decuplet with charge eQ the value $\mu = Q\mu_p$ (μ_p: proton magnetic moment) [9], resulting for the Δ^{++} in $\mu_{\Delta^{++}} = 5.58\,\mu_N$. The magnetic moment of the $\Delta^+(1232)$ is accessed in the reaction $\gamma p \rightarrow p\pi^0\gamma'$ which has been measured by the A2/TAPS Collaboration at MAMI [10]. Using theoretical input based on the phenomenological model of ref. [11] the extracted value reads $\mu_{\Delta^+} = (2.7^{+1.0}_{-1.3}(\text{stat.}) \pm 1.5(\text{syst.}) \pm 3(\text{theor.}))\,\mu_N$ [10] (see also refs. [12–16] for additional theoretical approaches to $\gamma p \rightarrow p\pi^0\gamma'$).

On the theoretical side, predictions for the delta magnetic moment have been obtained in various approaches such as $SU(6)$ symmetry [9], several quark models [17–28], the Skyrme model [29], the $1/N_c$ expansion [30], lattice QCD [31–33], QCD sum rules [34,35], heavy-baryon chiral perturbation theory (HBChPT) [32,36,37], quenched ChPT [38], and chiral effective field theory [16]. The aim of this letter is to calculate the magnetic moment of the $\Delta(1232)$ up to and including chiral order p^3 in a manifestly Lorentz-invariant formulation of baryon chiral perturbation theory with explicit Δ degrees of freedom (ΔChPT) [39][2]. Our approach differs from that of

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[1] μ_N denotes the nuclear magneton $e/(2m_p)$.
[2] Here, p stands for small parameters of the theory like the pion mass and the Δ-nucleon mass difference.

a previous manifestly Lorentz-invariant calculation [16] in the structure of the effective Lagrangian, the power-counting scheme, and the renormalization scheme. In sect. 2 we introduce the relevant effective Lagrangian and state the power counting. In sect. 3 we calculate the magnetic moment of the $\Delta(1232)$ at $\mathcal{O}(p^3)$. Section 4 contains a short summary.

2 Effective Lagrangian

The effective Lagrangian and the power counting relevant to classifying the renormalized diagrams for the calculation of the magnetic moment of the $\Delta(1232)$ have been discussed in ref. [39]. The non-resonant part of the effective Lagrangian is that of BChPT with only pion and nucleon fields [40] (see, e.g., refs. [41] for an introduction). All parameters and fields are considered as renormalized quantities in the extended on-mass-shell (EOMS) renormalization scheme of ref. [42]. The effective Lagrangian of the $\Delta(1232)$-resonance $[I(J^P) = \frac{3}{2}(\frac{3}{2}^+)]$ is formulated in terms of vector-spinor isovector-isospinor Rarita-Schwinger fields $\Psi_{\mu,i}$ [43]. The most general Lagrangian depends on a free "off-shell parameter" A [44]. Analyzing the constraints required for obtaining the correct number of physical degrees of freedom one finds that not all coupling constants of the original most general Lagrangian are independent [45]. The relations among the coupling constants also involve the parameter A, however, in such a way that the resulting effective Lagrangian is invariant under the set of "point transformations" (see, refs. [39] and [45] for further details). As a result of this invariance, physical quantities do not depend on A and we are free to choose a convenient value for A, say, $A = -1$.

For this choice of A, the leading-order Lagrangian reads

$$\mathcal{L}_\Delta^{(1)} = \bar{\Psi}_\mu \xi^{\frac{3}{2}} \Lambda^{\mu\nu} \xi^{\frac{3}{2}} \Psi_\nu, \qquad (1)$$

with the isospin projection operator $\xi_{ij}^{\frac{3}{2}} = \delta_{ij} - \frac{1}{3}\tau_i\tau_j$ and

$$\begin{aligned}
\Lambda_{\mu\nu} = -\Bigg\{ &(i\slashed{D} - m_\Delta)\, g_{\mu\nu} - i\,(\gamma_\mu D_\nu + \gamma_\nu D_\mu) \\
&+ i\,\gamma_\mu \slashed{D} \gamma_\nu + m_\Delta\, \gamma_\mu\gamma_\nu \\
&+ \frac{g_1}{2}\left[\slashed{u}\, g_{\mu\nu} - \gamma_\mu u_\nu - u_\mu \gamma_\nu + \gamma_\mu \slashed{u}\, \gamma_\nu \right]\gamma_5 \Bigg\}.
\end{aligned} \qquad (2)$$

The covariant derivative of the delta field is defined as

$$(D_\mu \Psi)_{\nu,i} \equiv \partial_\mu \Psi_{\nu,i} - 2i\epsilon_{ijk}\Gamma_{\mu,k}\Psi_{\nu,j} + \Gamma_\mu \Psi_{\nu,i} - iv_\mu^{(s)}\Psi_{\nu,i}$$

and involves the connection $\Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger] = \tau_k \Gamma_{\mu,k}$. The pion fields are contained in the unimodular unitary (2×2) matrix U with $u^2 = U$. In case of the electromagnetic interaction, we insert for the external fields $r_\mu = l_\mu = -e\frac{\tau_3}{2}\mathcal{A}_\mu$ and $v_\mu^{(s)} = -\frac{e}{2}\mathcal{A}_\mu$ [41], where e is the proton charge. The $\pi\Delta\Delta$ interaction is generated by the last term of eq. (2), where

$u_\mu = i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger] = \tau_k u_{\mu,k}$ and g_1 is the relevant coupling constant. Finally, m_Δ stands for the mass of the Δ. The leading-order $\pi N\Delta$ interaction Lagrangian reads

$$\mathcal{L}_{\pi N\Delta}^{(1)} = -g\,\bar{\Psi}_{\mu,i}\,\xi_{ij}^{\frac{3}{2}}\,(g^{\mu\nu} - \gamma^\mu\gamma^\nu)\,u_{\nu,j}\,\Psi + \text{h.c.}, \qquad (3)$$

where $\Psi = (p,n)^T$ denotes the nucleon field with two four-component Dirac fields p and n describing the proton and neutron, respectively, and g is a coupling constant. Finally, for the calculations of this work it is sufficient to parameterize the photon-delta interaction Lagrangian at $\mathcal{O}(p^2)$ as

$$\mathcal{L}_\Delta^{(2)} = \frac{ie}{2m_\Delta}\,\bar{\Psi}_\mu \xi^{\frac{3}{2}}\left[1 + \frac{3}{2}d_1 + 3\left(1 + \frac{3}{2}d_2\right)\tau_3\right]\xi^{\frac{3}{2}}\Psi_\nu\,\mathcal{F}^{\mu\nu}, \qquad (4)$$

where $\mathcal{F}^{\mu\nu}$ denotes the field-strength tensor and d_1, d_2 are coupling constants contributing to the magnetic dipole moment of the Δ at the given order[3].

The perturbative calculation of the dipole moment is organized by applying the following power counting to the renormalized diagrams. Interaction vertices obtained from an $\mathcal{O}(p^n)$ Lagrangian count as order p^n, a pion propagator as order p^{-2}, a nucleon propagator as order p^{-1}, and the integration of a loop as order p^4. In addition, we assign the order p^{-1} to the Δ propagator and the order p^1 to the mass difference $\delta \equiv m_\Delta - m$.

In a resonance generating channel, a Δ propagator which is not involved in a loop integration has to be dressed. One then has to re-sum the self-energy insertions and to consider the dressed propagator as of the order p^{-3}, because the self-energy starts at $\mathcal{O}(p^3)$.

3 Magnetic moment of the $\Delta(1232)$

Unstable particles do not occur in the spectrum of asymptotic states of the theory. Therefore the standard definition of the magnetic moment through the matrix element of the current between asymptotic free states cannot be applied. Instead one considers a complete physical scattering amplitude where the unstable particle contributes as an intermediate state. One parameterizes the contribution of the unstable particle and defines the magnetic moment such that for the regime where the unstable particle turns into a stable one (here $m_\Delta < m_N + M_\pi$) the magnetic moment coincides with the standard one of the stable particle.

For example, for the $\Delta^+(1232)$-resonance one considers the physical process

$$\gamma + p \to p + \pi^0 + \gamma'. \qquad (5)$$

In the Δ-resonance region, to leading orders (p^{-3}, p^{-2}, p^{-1}) in ΔChPT the contribution of the $\Delta^+(1232)$ can be

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[3] The separation has been introduced for later convenience so that the final expression of the magnetic moment in the usual isospin basis is most simple.
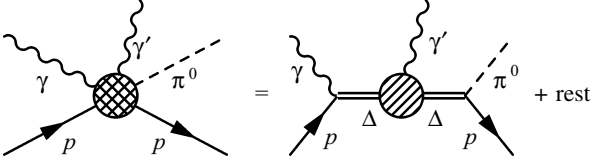
Fig. 1. $\gamma p \to p\pi^0 \gamma'$ amplitude in the $\Delta(1232)$-resonance region.

consistently separated. The contribution shown in fig. 1 is symbolically of the form

$$V_\delta \, S^{\delta\gamma} \, \Gamma^\nu_{\gamma\beta} \, S^{\beta\alpha} V^\mu_\alpha \,, \tag{6}$$

where V^μ, Γ^ν, and V denote the $\gamma N\Delta$, $\gamma\Delta\Delta$, and $\pi\Delta N$ vertices, respectively, and S corresponds to the Δ propagator. We parameterize the (most general) $\gamma\Delta\Delta$ vertex Γ^ν in terms of the Lorentz structures

$$\Gamma^\nu_{\gamma\beta} = g_{\gamma\beta}\Gamma^\nu_1(p',p) + \gamma_\gamma \Gamma^\nu_2(p',p)\gamma_\beta + p'_\gamma \Gamma^\nu_3(p',p)p_\beta + \cdots$$

and expand around $p^2 = m^2_\Delta, \not{p} = m_\Delta, p'^2 = m^2_\Delta, \not{p}' = m_\Delta$. Then, the only relevant contribution up to and including the next-to-next-to-leading order reads[4]

$$V_\delta \, S^{\delta\gamma} \, g_{\gamma\beta} \, \Gamma^\nu_1(p',p)\Big|_{p^2=m^2_\Delta, \not{p}=m_\Delta, p'^2=m^2_\Delta, \not{p}'=m_\Delta} \, S^{\beta\alpha} V^\mu_\alpha \,. \tag{7}$$

As a result of eq. (7), at leading orders (p^{-3}, p^{-2}, p^{-1}) one can consider the $\gamma\Delta\Delta$ vertex function with an "on-mass-shell Δ" and parameterize

$$\Gamma^\nu_1(p',p) = \gamma^\nu F(Q^2) + \frac{(p+p')^\nu}{2m_\Delta} G(Q^2) + \cdots,$$
$$q^\nu = (p'-p)^\nu, \qquad Q^2 = -q^2 \,, \tag{8}$$

where the complete on-shell vertex contains two additional structures [46] which are, however, not related to the magnetic moment. We express the total magnetic moment as

$$\vec{\mu} = [Q+\kappa] \, \frac{e}{2m_\Delta} \, g \, \vec{S}, \tag{9}$$

where $\vec{S}$ is the spin, eQ is the charge, and $g = 2$ (in combination with $\kappa = 0$) is the gyromagnetic ratio of a particle that does not participate in the strong interactions, neglecting also higher-order weak and electromagnetic interactions [47][5]. Here, we will only consider the modification due to the strong interactions which are encoded in the anomalous magnetic moment κ. The total magnetic moment in units of $e/(2m_\Delta)$ is given by $F(0)$ of eq. (8). Performing an isospin decomposition in the isovector-isospinor representation as

$$F(Q^2) = \frac{1}{2}F^{(s)}(Q^2) + \frac{3}{2}\tau_3 F^{(v)}(Q^2),$$

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[4] Note that $\gamma^\alpha S_{\alpha\beta}(p)$ and $p^\alpha S_{\alpha\beta}(p)$ are free of poles and therefore generate only terms of higher order.

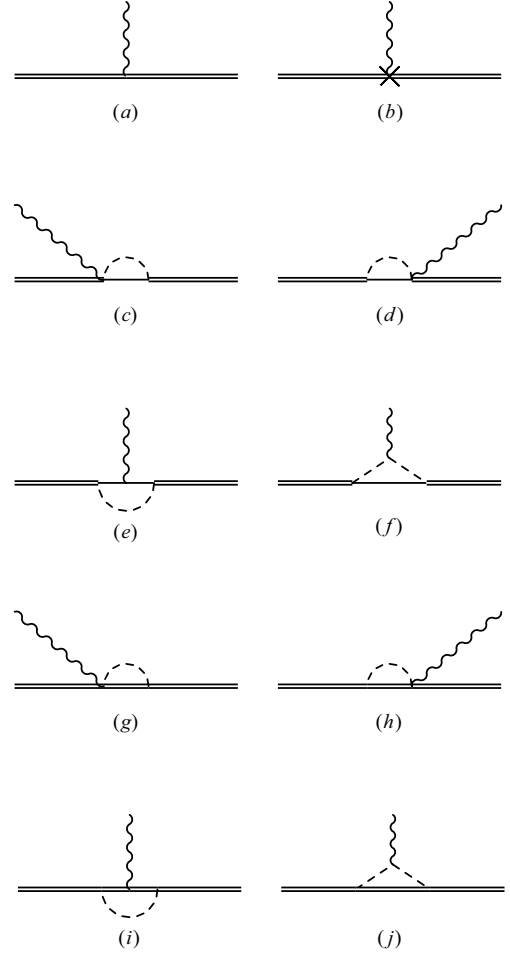[5] The use of minimal substitution only generates $g = 2/3$ instead of $g = 2$.

Fig. 2. Contributions to the $\gamma\Delta\Delta$ vertex up to and including $\mathcal{O}(p^3)$.

one obtains for the isoscalar and isovector components of the magnetic dipole moment

$$\mu^{(s)}_\Delta = F^{(s)}(0) \, \frac{e}{2m_\Delta}, \qquad \mu^{(v)}_\Delta = F^{(v)}(0) \, \frac{e}{2m_\Delta} \,. \tag{10}$$

The magnetic dipole moment of the physical degrees of freedom is given by

$$\mu = \frac{1}{2} \mu^{(s)}_\Delta + T_3 \, \mu^{(v)}_\Delta$$
$$= 3 \left[\frac{1}{2} \left(1 + \kappa^{(s)}_\Delta\right) + T_3 \left(1 + \kappa^{(v)}_\Delta\right) \right] \frac{e}{2m_\Delta} \,, \tag{11}$$

where T_3 stands for the third component of the isospin operator in the usual four-dimensional representation.

Using the Lagrangians of sect. 2, we have calculated the $\gamma\Delta\Delta$ vertex up to and including $\mathcal{O}(p^3)$, where the relevant diagrams are shown in fig. 2. Applying the EOMS renormalization scheme [42][6], we obtain the following renormalized expressions for the form factors $F^{(s)}$ and

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[6] The contribution of diagram (i) of fig. 2 to the magnetic moment is of higher order.

$F^{(v)}$ at $Q^2 = 0$:

$$F^{(s)}(0) = 3 + 3d_1 + \frac{71 g^2 m_\Delta \delta}{512\,\pi^2\,F^2} + \mathcal{O}(p^4),$$

$$\begin{aligned}
F^{(v)}(0) = {}&3 + 3d_2 - \frac{g_1^2 M m_\Delta}{54\,\pi\,F^2} \\
&- \frac{g^2 m_\Delta}{4608\,\pi^2\,F^2}\left[443\,\delta + 384\,\delta \,\ln\left(\frac{M}{m_\Delta}\right) \right. \\
&\left. + 384 \Theta \,\ln\left(\frac{\delta + \Theta}{M}\right) \right] + \frac{i\,g^2 m_\Delta \Theta}{12\,\pi\,F^2} \\
&+ \mathcal{O}(p^4),
\end{aligned} \tag{12}$$

where $\Theta = \sqrt{-M^2 + \delta^2}$. For the numerical analysis we make use of $g_A = 1.267$, $F_\pi = 92.4\,\mathrm{MeV}$, $m_N = m_p = 938.3\,\mathrm{MeV}$, $M_\pi = M_{\pi^+} = 139.6\,\mathrm{MeV}$, and $m_\Delta = 1210\,\mathrm{MeV}$, where m_Δ is the pole mass. For the Δ coupling constants we take $g = 1.127$ as obtained from a fit to the $\Delta \to \pi N$ decay width [39] and $g_1 = 9 g_A/5$ from $SU(6)$ symmetry. We then obtain for $\kappa^{(s)}$ and $\kappa^{(v)}$

$$\begin{aligned}
\kappa_\Delta^{(s)} &= d_1 + 0.23 + \mathcal{O}(p^4), \\
\kappa_\Delta^{(v)} &= d_2 - 0.22 + i\,0.37 + \mathcal{O}(p^4).
\end{aligned} \tag{13}$$

Unfortunately, we do not have an estimate for the parameters d_1 and d_2 which reflect the contribution to the anomalous magnetic moment of the $\Delta(1232)$ at tree level [$\mathcal{O}(p^2)$]. However, we can compare the results of eq. (13) with the anomalous magnetic moment of the nucleon [48]

$$\begin{aligned}
\kappa_N^{(s)} &= 2 c_7 m + \mathcal{O}(p^4), \\
\kappa_N^{(v)} &= 4 c_6 m - \frac{g_A^2 m M}{4\pi F^2} = 4 c_6 m - 1.96 + \mathcal{O}(p^4),
\end{aligned} \tag{14}$$

where c_6 and c_7 are parameters of the $\mathcal{O}(p^2)$ πN Lagrangian. There are two main differences between the anomalous magnetic moments of the nucleon and of the $\Delta(1232)$ up to the chiral order p^3. First, pion loops do not contribute to $\kappa_N^{(s)}$ at $\mathcal{O}(p^3)$, whereas the loop contribution to $\kappa_\Delta^{(s)}$ is 0.23. The loop contribution originates from the renormalized diagram (e) of fig. 2. Second, due to the unstable nature of the $\Delta(1232)$, there is an imaginary part in $\kappa_\Delta^{(v)}$ which the nucleon does not have. The isovector loop contribution is significantly smaller than in the case of the nucleon.

Let us finally compare the results with previous EFT calculations. As in the nucleon case, ChPT in the $SU(2)$ sector does not *predict* the anomalous magnetic moments of the $\Delta(1232)$. In refs. [32,36,37] the decuplet magnetic moments were calculated in the heavy-baryon framework. Our calculation differs by the set of diagrams which contibute at the given order, because we also include the renormalized diagram (e) of fig. 2. The manifestly Lorentz-invariant approach of ref. [16] uses a different power counting and considers a different set of diagrams. A *numerical* comparison of the various EFT approaches would require knowledge of the low-energy coupling constants. Unfortunately, an estimate of these coupling constants does

not exist yet. Finally, note that different renormalization schemes result in different values for the low-energy constants.

4 Summary

We have calculated the magnetic dipole moment of the $\Delta(1232)$ up to and including order p^3 treating both the pion mass and the Δ-nucleon mass difference as small quantities of order p. For this purpose we have used the manifestly Lorentz-invariant form of BChPT with explicit Δ degrees of freedom [39] in combination with the EOMS renormalization scheme [42]. This results in a consistent effective field theory describing the correct number of physical degrees of freedom in combination with a systematic power counting. The $\pi\Delta\Delta$ interaction was chosen to be consistent with a recent analysis of the structure of constraints of ref. [45] for a spin-(3/2) system.

At next-to-leading order, $\mathcal{O}(p^2)$, the isoscalar and isovector anomalous magnetic moments are given in terms of two low-energy constants. At next-to-next-to-leading order the isoscalar anomalous magnetic moment receives a real loop contribution of 0.18 in units of the nuclear magneton. This has to be contrasted with the nucleon, where the loop contribution to the isoscalar anomalous magnetic moment is $\mathcal{O}(p^4)$. At next-to-next-to-leading order the isovector anomalous magnetic moment receives a real loop contribution of -0.17 and an imaginary loop contribution of 0.29 in units of μ_N. The appearence of an imaginary part in the $\gamma\Delta\Delta$ vertex function reflects the unstable nature of the $\Delta(1232)$. Our results differ from previous EFT approaches in the structure of the effective Lagrangian, the power-counting scheme, and the renormalization scheme.

As a next step it would be desirable to have full and consistent calculations of $\pi^+ p$ bremsstrahlung and $\gamma p \to p\pi^0 \gamma'$ in the Δ-resonance region. Such calculations would have the potential of allowing for an extraction of the parameters d_1 and d_2 from a fit to the experimental cross-sections and thus for obtaining a result for the magnetic moments in a self-consistent framework.

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